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# The Crystal Structure of $\boldsymbol{\beta}-\mathbf{C a}_{2} \mathbf{P}_{\mathbf{2}} \mathrm{O}_{7}$ 

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Crystals of $\beta$ - $\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ are tetragonal, space group $P 4_{1}$, with 8 formula units in a unit cell with $a=b=$ $6 \cdot 684, c=24 \cdot 144 \AA$. Each of the two independent pyrophosphate groups is in an essentially eclipsed configuration. The mean values for the bond lengths in the pyrophosphate groups are $\mathrm{P}-\mathrm{O}$ (bridge) $=$ $1.615 \pm 0.017 \AA$ and $\mathrm{P}-\mathrm{O}=1.518 \pm 0.021 \AA$, and the $\mathrm{P}-\mathrm{O}-\mathrm{P}$ angles are 131 and $138^{\circ}$. Of the four independent calcium atoms two have seven nearest neighbor oxygen atoms, one has eight and one has nine. The structure exhibits a local, non-crystallographic center of symmetry.

## Introduction

One of the objectives of the determination of the structure of sodium pyrophosphate decahydrate (MacArthur \& Beevers, 1957; Cruickshank, 1964) was to establish the geometry of the pyrophosphate ion and, in particular, to determine the $\mathrm{P}-\mathrm{O}-\mathrm{P}$ angle. The $\mathrm{P}-\mathrm{O}-\mathrm{P}$ angle was found to be $134^{\circ}$, which is quite different from the value of $180^{\circ}$ reported in cubic $\mathrm{ZrP}_{2} \mathrm{O}_{7}$ by Levi \& Peyronel (1935). In $\mathrm{ZrP}_{2} \mathrm{O}_{7}$ the central oxygen atom lies on a center of symmetry, and in $\mathrm{Na}_{4} \mathrm{P}_{2} \mathrm{O}_{7} \cdot 10 \mathrm{H}_{2} \mathrm{O}$ it lies on a twofold axis. In the $\beta$ phase of calcium pyrophosphate, $\beta-\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$, all atoms are in general positions, thereby providing an opportunity for studying the geometry of pyrophosphate groups free of symmetry restrictions. In other investigations of structures containing a pyro-group, $\mathrm{X}_{2} \mathrm{O}_{7}$, the $\mathrm{X}-\mathrm{O}-\mathrm{X}$ angle has been found to be $128^{\circ}$ in $\mathrm{Cl}_{2} \mathrm{O}_{7}$ (Fonteyne, 1938), $124^{\circ}$ in $\mathrm{K}_{2} \mathrm{~S}_{2} \mathrm{O}_{7}$ (Lynton \& Truter, 1960) and $180^{\circ}$ in $\mathrm{Sc}_{2} \mathrm{Si}_{2} \mathrm{O}_{7}$ (Zachariasen, 1930; Cruickshank, Lynton \& Barclay, 1962). In $\beta-\mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ and $\beta-\mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ the $\mathrm{P}-\mathrm{O}-\mathrm{P}$ group appears to be linear because of a thermal averaging of the position of the central oxygen atom (Calvo, 1965a, b).

## Experimental

Heating dicalcium phosphate dihydrate to approximately $750^{\circ} \mathrm{C}$ produces $\beta-\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ which melts at approximately $1350^{\circ} \mathrm{C}$ :

$$
\begin{aligned}
& 2 \mathrm{CaHPO}_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O} \stackrel{\sim 120^{\circ} \mathrm{C}}{-4 \mathrm{H}_{2} \mathrm{O}} 2 \mathrm{CaHPO}_{4} \\
& \sim 450^{\circ} \mathrm{C} \rightarrow \gamma-\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7} \stackrel{\sim}{\sim}-\frac{0^{\circ} \mathrm{C}}{\rightarrow} \beta-\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7} \\
& \sim-\mathrm{H}_{2} \mathrm{O}
\end{aligned}
$$

A sample of $\mathrm{CaHPO} 4.2 \mathrm{H}_{2} \mathrm{O}$ in a Vycor crucible was placed in a muffle oven and heated to $\sim 1400^{\circ} \mathrm{C}$. The oven was turned off but left closed to allow slow cooling of the melt. The solidified melt obtained in this way had a glassy appearance, but on shattering, the melt yielded small single crystals in the shape of rectangular solids. One of these crystals ( $0.25 \times 0.17 \times 0.10 \mathrm{~mm}$ ) was mounted with its long axis (later found to be $a$ ) coincident with the axis of rotation.

Corbridge (1957) reported $\beta-\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ to crystallize in the tetragonal space group $P 4_{1}$. We observed systematic absences $(00 l, l \neq 4 n)$ on Weissenberg photographs indicating either space group $P 4_{1}$ or $P 4_{1} 22$, but the relationship observed in the intensity data, $|F(h k l)|$ $\neq|F(h \bar{k} l)|$, eliminated $P 4_{1} 22$ as a possibility. Precise values of the cell parameters $b$ and $c$ were obtained by least-squares refinement on data for seventeen 0 kl reflections taken from measurements of a Straumanistype rotation photograph. In adjusting the cell parameters to give the best fit between observed and calculated $\sin ^{2} \theta$ values no allowance was made for eccentricity or absorption effects; each observational equation was weighted with $V / w=1 / \sin 2 \theta$, and $\lambda\left(\mathrm{Cu} K \alpha_{1}\right)$ was taken as $1.5405 \AA$. The results obtained here are listed in Table 1 along with those obtained by Corbridge (1957) and Keppler (1962); the standard deviations
estimated from the least-squares treatment seemed unrealistically small and hence were multiplied by threc before listing in Table 1. (Multiplying by three raises the confidence level to $99 \%$ ). The density was measured by the flotation method with solutions of carbon tetrachloride and tin tetrabromide; the calculated densities are based on eight units of $\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ per unit cell.

Table 1. Unit-cell dimensions and densities of $\beta-\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ as found by Corbridge (1957) and Keppler (1962) and as determined in this work

|  | Corbridge | Keppler | This work |
| :--- | :---: | ---: | ---: |
| $a(\AA)$ | $6 \cdot 66$ | $6 \cdot 685 \pm 0.01$ | $6 \cdot 684 \pm 0.006$ |
| $c(\AA)$ | $23 \cdot 86$ | $24 \cdot 154 \pm 0.05$ | $24 \cdot 144 \pm 0.015$ |
| $\varrho_{o}\left(\mathrm{~g} . \mathrm{cm}^{-3}\right)$ | $3 \cdot 1$ | $3 \cdot 119 \pm 0.01$ | $3 \cdot 120 \pm 0.005$ |
| $\varrho_{c}\left({\left.\mathrm{~g} . \mathrm{cm}^{-3}\right)}\right.$ | $3 \cdot 19$ | $3 \cdot 127$ | $3 \cdot 128$ |

Weissenberg photographs were recorded for the zero through fifth layer lines about a using $\mathrm{Cu} K \alpha$ radiation and employing the equi-inclination multiple-film technique. A striking feature of the photographs was that there were intense reflections all the way to the edge of the film. Intensities were estimated by visual comparison with a calibrated strip prepared from one reflection of the crystal used throughout this investigation. Intensities were corrected for Lorentz-polarization and absorption effects. The absorption corrections were made with the aid of a program for the IBM 1620 computer written by Craven (1963), who used a procedure suggested by Busing \& Levy (1957). In practice, the absorption corrections were not made until the need was indicated in the refinement process; when the need became apparent, the raw data were reprocessed. Corrected intensities from different layers were correlated by using the relationship $|F(h \bar{k} l)|=|F(k h l)|$. In all, 1150 independent reflections $(89 \%$ ) were estimated out of a possible 1296 in the $\mathrm{Cu} K \alpha$ sphere. Of the 1150 total estimated reflections, 80 were estimated as being less than a minimum observable intensity.

In all of the structure-factor calculations the scattering factors used were for $\mathrm{Ca}^{2-}, \mathrm{P}$ and an average of O and $\mathrm{O}^{-}$as given in International Tables for $X$-ray Crystallography (1962).

## Structure determination

With space group $P 4_{1}$ and 8 units of $\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ in the unit cell there must be 4 Ca and 4 P atoms in the asymmetric unit. The positions of these heavy atoms were deduced from a three-dimensional Patterson map, relying heavily on the Harker sections at $w=\frac{1}{4}$ and $\frac{1}{2}$, and from electron-density maps calculated on the basis of 4 out of 8 and then 6 out of 8 of the heavy atoms. The trial model for the heavy atoms was derived with some scepticism because the eight heavy atoms occur in pairs in four layers in the $\mathbf{c}$ direction and the vector relating the atoms in any one pair is essentially the same as that relating the atoms within the other pairs. Nearly identical $\mathbf{C a}-\mathbf{P}$ vectors also made me feel uneasy
about the trial solution. An early clue to the layered nature of the heavy atoms was the high intensity of the 008 reflection, which indicated layers separated by approximately $\frac{1}{8}$ along $\mathbf{c}$; this indication is substantiated by the final results.

After location of the heavy atoms the oxygen atoms were found in two subsequent Fourier syntheses. Expected $\mathrm{P}-\mathrm{O}$ distances were consulted in making judicious choices of oxygen positions from the excess of possible positions indicated in the electron-density maps. From the first map 10 of the 14 oxygen atoms were located, and while the positions of the remaining oxygen atoms could be predicted, the electron densities at these positions were barely above background, and another Fourier synthesis was carried out with the 4 oxygen atoms omitted; in the resulting map the missing atoms were readily apparent.

## The refinement

A structure-factor calculation based on the trial positions of all eight heavy atoms showed reasonably good agreement between observed and calculated values as indicated by $R=0.35$, where $R$ is the usual agreement index. (In the early stages of seeking a solution only the low order $60 \%$ of the data were used.) After a complete trial structure had been obtained and with unit weights and estimated fixed temperature factors, three cycles of least-squares brought $R$ to $0 \cdot 19$. At this stage some of the observed $\mathrm{P}-\mathrm{O}$ distances were not close to expected values. Consequently five of the oxygen atoms were shifted to give satisfactory $\mathrm{P}-\mathrm{O}$ distances, and the next structure-factor calculation showed an agreement index of 0.147 which dropped to $0 \cdot 127$ in three cycles. At this point individual isotropic temperature factors were introduced into the refinement over all of the data rather than just the low order $60 \%$. The first structure-factor calculation showed $R=0 \cdot 135$ and a few least-squares cycles lowered $R$, but all of the heavy atoms and some of the oxygen atoms had negative temperature factors. By comparing lists of $F_{o}$ 's and $F_{c}$ 's it was apparent that the data were suffering from absorption effects. As indicated above, absorption corrections were made and the first structurefactor calculation showed $R=0.095$. In all subsequent refinement the weighting function (similar to Hughes, 1941) was taken as

$$
\begin{aligned}
& V w^{\prime}=\frac{1}{S\left(F_{o}+K\right)} \text { for } F_{o}>4 F_{\min } \\
& \quad \text { and } V^{\prime} w=\frac{1}{S\left(4 F_{\min }+K\right)} \text { for } F_{o} \leq 4 F_{\min }
\end{aligned}
$$

where $4 F_{\min }=31 \cdot 7$ and $K=7 \cdot 9$. Usually the value of $S$ was inversely proportional to the square root of the number of times the intensity was estimated, with slight variations depending on a subjective evaluation of the accuracy of the estimation. $S$ was assigned a much larger value than usual for the 008 reflection because it probably suffers from extinction, and for the 004

Table 2．Observed and calculated structure factors
The five columns within each group contain the values $l, 10 F_{o}, 10 F_{c}, 10 A_{c}$ and $10 B_{c}$ ．











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reflection because it was near the beam stop. Throughout, full-matrix least-squares refinement cycles were used in minimizing the quantity $\Sigma w\left(\left|F_{o}\right|-\left|F_{c}\right|\right)^{2}$. Three cycles in which all the positional parameters, isotropic temperature factors and the scale factor were varied reduced $R$ to 0.068 ; in the last two cycles the shifts were less than the standard deviations. Two additional cycles did not reduce $R$ and the shifts were all less than one-fifth the estimated standard deviations for the coordinates and all less than one-third for the temperature factors. The final fractional coordinates and isotropic temperature factors are listed in Table 3. The above value for $R$ does not include the 'less-than' reflections; out of the total of 80 'less-thans', only 3 had $F_{c}>F_{\min }$ and none of these by more than $9 \%$. If the 008 reflection, which appears to be suffering from extinction, is disregarded, the agreement index is 0.065 .

In the final electron-density map, in which the 'lessthan' reflections were given half weight, the ranges in peak height in electrons per cubic $\AA$ are 61 to 65 for


Fig. 3. An illustration of the infinite chains based on the chelate-like rings. This drawing shows the chain formed by pyrophosphate group I $[\mathrm{P}(1)$ and $\mathrm{P}(2)$ ] and $\mathrm{Ca}(3)$ and $\mathrm{Ca}(4)$. The view is along $\mathbf{b}$.
$\mathrm{Ca}, 49$ to 52 for P and 18 to 20 for O , and the extremes in background are found in ripples around the heavy atoms and ranged from $5 \mathrm{e} . \AA^{-3}$ to $-5 \mathrm{e} . \AA^{-3}$; the ripples are not symmetric and probably are indicative of anisotropic thermal motion of the heavy atoms.

## Description of the structure

One layer of the structure is illustrated in Fig. 1; a few oxygen atoms have been added where necessary to show complete coordination shells around the Ca atoms. The heavy atoms occur in pairs of like atoms in planes at $z=0,0.02,0.12$ and 0.14 in the order Ca , $\mathrm{P}, \mathrm{P}$, and Ca . Oxygen atoms of the pyrophosphate groups extend above and below the sheets of Ca atoms to provide bonding between adjacent sheets. Packed around each Ca atom are $6 \mathrm{PO}_{4}$ tetrahedra with P atoms at the vertices of a distorted octahedron. There is a pseudo-center of symmetry at $x=0.456, y=0.520$ and $z=0.067$, which are the mean values of the coordinates of centers relating $\mathrm{Ca}(1)-\mathrm{Ca}(4), \mathrm{Ca}(2)-\mathrm{Ca}(3)$, $\mathrm{P}(1)-\mathrm{P}(4), \mathrm{P}(2)-\mathrm{P}(3), \mathrm{O}(5)-\mathrm{O}(8)$ and so on for all appropriate pairs; the ranges in values are $0.44-0.47$, $0.51-0.53$ and $0.063-0.074$. This pseudo-center holds within one layer of the structure as shown in Fig. 1, but atoms above a layer do not have centrosymmetrically related atoms below the layer. That is, the fourfold screw symmetry does not preserve the center.

Each pyrophosphate group is linked in all directions to surrounding groups by commonly shared Ca atoms. As one might expect, there are infinite chains running throughout the structure. In Fig. 3 is illustrated a pyro-phosphate-Ca chain running parallel to a and perpendicular to the chains which are above and below and are parallel to $\mathbf{b}$, giving a matted effect when viewed down c. Within a layer there are two such chains parallel to each other, being related by the pseudo-

Table 3. Fractional atomic coordinates with standard deviations ( $\times 10^{4}$ ) in parentheses and isotropic temperature factors with standard deviations $\left(\times 10^{2}\right)$ in parentheses

|  | $x$ | $y$ | $z$ | $B\left(\AA^{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Ca}(1)$ | $-0 \cdot 2101$ (4) | $0 \cdot 2656$ (4) | $0 \cdot 0$ | $0 \cdot 35$ (4) |
| $\mathrm{Ca}(2)$ | $0 \cdot 5449$ (4) | $0 \cdot 1583$ (4) | $0 \cdot 1454$ (1) | $0 \cdot 55$ (4) |
| $\mathrm{Ca}(3)$ | $0 \cdot 3619$ (4) | -0.1061 (4) | $0 \cdot 0026$ (1) | $0 \cdot 50$ (4) |
| $\mathrm{Ca}(4)$ | $0 \cdot 1391$ (4) | -0.2314 (4) | $0 \cdot 1379$ (1) | $0 \cdot 56$ (4) |
| P (1) | -0.1327 (5) | $0 \cdot 7449$ (5) | $0 \cdot 0134$ (1) | $0 \cdot 26$ (4) |
| $\mathrm{P}(2)$ | -0.3782 (5) | 0.6929 (5) | $0 \cdot 1141$ (1) | 0.38 (5) |
| $\mathrm{P}(3)$ | $0 \cdot 2893$ (5) | $0 \cdot 3544$ (5) | 0.0171 (1) | $0 \cdot 43$ (5) |
| $\mathrm{P}(4)$ | $0 \cdot 0467$ (5) | $0 \cdot 2850$ (5) | $0 \cdot 1194$ (2) | $0 \cdot 42$ (4) |
| $\mathrm{O}(1)$ | -0.1012 (13) | $0 \cdot 5912$ (13) | -0.0313 (3) | $0 \cdot 52$ (13) |
| O(2) | -0.2615 (13) | 0.9166 (14) | -0.0096 (4) | $0 \cdot 63$ (13) |
| O(3) | 0.0529 (13) | $0 \cdot 8224$ (13) | 0.0427 (3) | $0 \cdot 63$ (13) |
| $\mathrm{O}(4)$ | -0.2612 (14) | $0 \cdot 6186$ (14) | $0 \cdot 0593$ (4) | $0 \cdot 86$ (14) |
| O(5) | $0 \cdot 5338$ (15) | $0 \cdot 5072$ (17) | $0 \cdot 1369$ (4) | $1 \cdot 30$ (16) |
| O(6) | $0 \cdot 4610$ (14) | 0.8407 (14) | 0.0951 (4) | $0 \cdot 92$ (15) |
| O(7) | -0.2256 (13) | 0.7986 (14) | $0 \cdot 1509$ (4) | $0 \cdot 60$ (13) |
| O(8) | $0 \cdot 3460$ (13) | 0.5469 (13) | -0.0105 (3) | 0.64 (14) |
| $\mathrm{O}(9)$ | $0 \cdot 4627$ (14) | $0 \cdot 2274$ (14) | 0.0356 (4) | $0 \cdot 87$ (14) |
| O(10) | $0 \cdot 1481$ (14) | $0 \cdot 2252$ (14) | -0.0201 (4) | 0.94 (15) |
| O(11) | $0 \cdot 1595$ (14) | $0 \cdot 4055$ (15) | 0.0704 (4) | 0.78 (15) |
| O(12) | $0 \cdot 0222$ (15) | $0 \cdot 4458$ (14) | $0 \cdot 1624$ (4) | $0 \cdot 89$ (14) |
| O(13) | $0 \cdot 1892$ (15) | $0 \cdot 1198$ (15) | $0 \cdot 1382$ (4) | 1.01 (15) |
| O(14) | -0.1492 (14) | $0 \cdot 2105$ (14) | $0 \cdot 0942$ (4) | $0 \cdot 74$ (14) |



Fig. 1. Illustration of the structure of $\beta-\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ viewed along $\mathbf{c}$, but including the cell contents between approximately $z=0$ and $z=0 \cdot 20$ only. The complete coordination shells around the four independent calcium ions are shown only once; these shells contain some oxygen atoms which are either above or below the general level shown.


Fig. 2. The arrangement of calcium atoms around pyrophosphate groups I and II as viewed along c. Group I and its cluster of calcium atoms have been inverted through the pseudo-center. See text.
center. Around a fourfold screw axis each set of pyrophosphate groups forms an infinite spiral held together by ionic bonds to Ca atoms.

The shortest $\mathrm{Ca}-\mathrm{Ca}$ distance is $3.69 \AA, \mathrm{Ca}(3)-\mathrm{Ca}(4)$. In $\mathrm{CaHPO}_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$ (Beevers, 1958) there are $\mathrm{Ca}-\mathrm{Ca}$ distances of $3.8 \AA$. The four shortest intermolecular $\mathrm{O}-\mathrm{O}$ distances in $\beta-\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ are $\mathrm{O}(12)-\mathrm{O}\left(2^{\prime}\right)=2.75 \AA$, $\mathrm{O}(5)-\mathrm{O}\left(8^{\prime}\right)=2.76 \AA, \mathrm{O}(6)-\mathrm{O}(13)=2.80 \AA$ and $\mathrm{O}(13)-$ $\mathrm{O}\left(1^{\prime}\right)=2 \cdot 85 \AA$.

## The pyrophosphate groups

Viewed down the $\mathrm{P}-\mathrm{P}$ axis, each crystallographically independent group is essentially in an eclipsed configuration as shown in Fig.4, whereas the group in $\mathrm{Na}_{4} \mathrm{P}_{2} \mathrm{O}_{7} \cdot 10 \mathrm{H}_{2} \mathrm{O}$ is staggered. Group I has eleven Ca atoms in its first shell of interaction and group II has ten. All Ca atoms considered to be within the first shell of a group are within $2 \cdot 65 \AA$ of at least one oxygen atom of that group. In order to raise the number of Ca atoms in the first shell around group II to eleven, one would have to consider a $\mathrm{Ca}-\mathrm{O}$ distance of $3.67 \AA$ as constituting a significant interaction. The clustering of Ca atoms around each of the groups is shown in Fig. 2; in that figure group I has been inverted through the pseudo-center to bring out the structural similarities between the two groups. Each group forms two chelatelike rings which demand an eclipsed configuration, and surprisingly, perhaps, the bridge oxygen atoms take part in coordination of the calciums. The bridge oxygen atoms $O(4)$ and $O(11)$, and $O(5)$ are bonded to only one Ca atom each, five oxygen atoms are bonded to two Ca atoms each, and six are bonded to three (here only $\mathrm{Ca}-\mathrm{O}$ distances of less than $3 \cdot 10 \AA$ have been considered as constituting a bond or as indicating a significant degree of coordination - this will be discussed later).

The bond distances in the pyrophosphate groups are shown in Fig. 5. If the pyrophosphate groups had ideal mm symmetry, there would be three structurally independent $\mathrm{P}-\mathrm{O}$ bonds. On looking at the distances in Fig. 5 it is obvious that the $4 \mathrm{P}-\mathrm{O}_{B}\left(\mathrm{O}_{B}\right.$ designates a bridge oxygen atom) bonds are longer than the others, and it is suggested that the $4 \mathrm{P}-\mathrm{O}$ bonds that are essentially coplanar with their respective $\mathrm{P}-\mathrm{O}_{B^{-}} \mathrm{P}$ bridges are shorter than the other $8 \mathrm{P}-\mathrm{O}$ bonds. The distances
for these three types of bond are $1.497 \pm 0.011 \AA$, $1.529 \pm 0.016 \AA$ and $1.615 \pm 0.019 \AA$; the standard deviation based on uncertainties in the least-squares coordinates is approximately $\pm 0.010 \AA$ for all $\mathrm{P}-\mathrm{O}$ bonds. If the first two groups are put together we find the $\mathrm{P}-\mathrm{O}$ distance to be $1.518 \pm 0.021 \AA$; the mean value for all $\mathrm{P}-\mathrm{O}$ bonds is $1.543 \bar{\AA}$, which is not different from the $1 \cdot 545 \AA$ predicted by Cruickshank (1961). It may be that the bonds in the $\mathrm{P}-\mathrm{O}_{B^{-}} \mathrm{P}$ plane are slightly contracted by enhanced delocalization and overlap through the oxygen $2 p \pi$ and phosphorus $3 d$ orbitals of the atoms in the plane. Those bonds may also be shorter because, in general, judging from the number and length of the $\mathrm{Ca}-\ldots \mathrm{O}$ bonds the in-plane oxygen atoms are not as strongly bonded to neighboring calcium atoms as are the other oxygen atoms.

The oxygen-to-oxygen distances within each pyrophosphate group are shown in Fig.6. In the same way that we have considered categorical differences in the $\mathrm{P}-\mathrm{O}$ distances we may select groups of $\mathrm{O}-\mathrm{O}$ distances and of $\mathrm{O}-\mathrm{P}-\mathrm{O}$ angles (Table 4). Within $\mathrm{PO}_{4}$ tetrahedra it seems that the distances between the bridge oxygen atom and the oxygen atoms in the $\mathrm{P}-\mathrm{O}_{B}-\mathrm{P}$ plane $(2.450 \pm 0.037 \AA)$ are shorter than the remaining distances $(2.527 \pm 0.032 \AA)$; the mean expected standard deviation for an $\mathrm{O}-\mathrm{O}$ distance is $\pm 0.013 \AA$. On examining the angles and distances one gets a picture of $\mathrm{PO}_{4}$ groups distorted from perfect tetrahedra by flaring of the oxygen atoms in the faces opposite the bridge atoms, that is, by a flattening of those $\mathrm{PO}_{3}$ pyramids. In both $\beta-\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ and $\mathrm{Na}_{4} \mathrm{P}_{2} \mathrm{O}_{7} \cdot 10 \mathrm{H}_{2} \mathrm{O}$ the $\mathrm{O}-\mathrm{P}-\mathrm{O}$

(I)

(II)

Fig. 4. A view of each pyrophosphate group along its $\mathbf{P}-\mathbf{P}$ axis. For group I, P(1) is toward the viewer; for group II, P(3).

Table 4. Oxygen-phosphorus-oxygen angles
e.s.d. $\pm 0 \cdot 5^{\circ}$ *

|  |  |  |  |
| :--- | :--- | :--- | :--- |
| $\mathrm{O}-\mathrm{P}-\mathrm{O}$   <br> $4-1-1$ $101 \cdot 9^{\circ}$ $11-3-8$ | $108 \cdot 3^{\circ}$ |  |  |
| $4-2-5$ | $103 \cdot 8$ | $11-4-12$ | $101 \cdot 6$ |
| $103 \cdot 9^{\circ} \pm 3 \cdot 1^{\circ}$ |  |  |  |
| $4-1-2$ | $109 \cdot 6^{\circ}$ | $11-3-9$ | $107 \cdot 5^{\circ}$ |
| $4-1-3$ | $106 \cdot 7$ | $11-3-10$ | $104 \cdot 7$ |
| $4-2-6$ | $106 \cdot 9$ | $11-4-13$ | $106 \cdot 6$ |
| $4-2-7$ | $107 \cdot 2$ | $11-4-14$ | $105 \cdot 8$ |
| $106 \cdot 9^{\circ} \pm 1 \cdot 4^{\circ}$ |  |  |  |


| $1-1-2$ | $109 \cdot 2^{\circ}$ | $8-3-9$ | $115 \cdot 0^{\circ}$ |
| :--- | :--- | ---: | :--- |
| $1-1-3$ | $116 \cdot 9$ | $8-3-10$ | $111 \cdot 9$ |
| $2-1-3$ | $111 \cdot 8$ | $9-3-10$ | $108 \cdot 9$ |
| $5-2-6$ | 111.9 | $12-4-13$ | $112 \cdot 3$ |
| $5-2-7$ | $113 \cdot 4$ | $12-4-14$ | $114 \cdot 5$ |
| $6-2-7$ | $110 \cdot 2$ | $13-4-14$ | $115 \cdot 0$ |

[^0]

Fig. 5. Bond distances and $\mathbf{P}-\mathrm{O}-\mathrm{P}$ angles in the pyrophosphate groups.


Fig.6. Oxygen to oxygen distances in the pyrophosphate groups.
angles are $113^{\circ}$ as compared with $105^{\circ}$ for the $\mathrm{O}-\mathrm{P}-\mathrm{O}_{B}$ angles. In the $\mathrm{S}_{2} \mathrm{O}_{7}^{2-}$ anion the structurally similar angles are 114 and $105^{\circ}$, and this distortion has been attributed to repulsion between the charged oxygen atoms (Lynton \& Truter, 1960). In Table 5 are some of the dimensions of the pyrophosphate group as found here and as found by Cruickshank in $\mathrm{Na}_{4} \mathrm{P}_{2} \mathrm{O}_{7} \cdot 10 \mathrm{H}_{2} \mathrm{O}$.

Table 5. Comparison of dimensions of pyrophosphate group as found in $\mathrm{Na}_{4} \mathrm{P}_{2} \mathrm{O}_{7} \cdot 10 \mathrm{H}_{2} \mathrm{O}$ with those found in this work

|  | $\mathrm{Na}_{4} \mathrm{P}_{2} \mathrm{O}_{7} .1 \mathrm{OH}_{2} \mathrm{O}$ | $\beta-\mathrm{Ca}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ <br> $\mathrm{P}-\mathrm{O}_{\text {outer }}$ |
| :--- | :--- | :--- |
| $1.51 \pm 0.015 \AA$ | $1.518 \pm 0.021 \AA$ |  |
| $\mathrm{P}-\mathrm{O}_{\text {Bridge }}$ | $1.61 \pm 0.015$ | $1.615 \pm 0.017$ |
| $\mathrm{P}-\mathrm{O}_{B}-\mathrm{P}$ | $133 \cdot 6^{\circ}$ | $134 \cdot 2^{\circ}$ |
| $\mathrm{O}_{B}-\mathrm{P}-\mathrm{O}$ | 105 | $105 \cdot 4$ |
| $\mathrm{O}-\mathrm{P}-\mathrm{O}$ | 113 | 112.6 |

Pyrophosphate group II is distorted; the $\mathrm{O}(10)-$ $\mathrm{O}(14)$ separation ( $3 \cdot 401 \AA$ ) is significantly larger than the other three of its kind $(\sim 3 \cdot 18 \AA)$. This is because $\mathrm{Ca}(1)$ is jammed into the side of the anion - as indicated by its approach to the bridge oxygen, $3 \cdot 14 \AA$, which is close in comparison to the three other similar $\mathrm{Ca}-\mathrm{O}_{B}$ approaches of $3.41,3.43$ and $3.56 \AA$. In addition, the two pyrophosphate groups significantly differ in their $\mathrm{P}-\mathrm{P}$ separation, 2.955 and $2.991 \AA$, both $\pm 0.005 \AA$; this is, of course, reflected in the $\mathrm{P}-\mathrm{O}_{B}-\mathrm{P}$ angles of 130.5 and $137 \cdot 8^{\circ}$, both $\pm 0 \cdot 8^{\circ}$. The differences and distortions are probably due to differences in unsymmetrical environments.

## Calcium coordination

Of the four independent calcium atoms in the structure two exhibit a coordination number of 7 , one of 8 and one of 9 . These coordination numbers reflect a judgment of which oxygen atoms are in the first shell, that is, the nearest neighbors. For these nearest neighbors the longest $\mathrm{Ca}-\mathrm{O}$ distance is $2.93 \AA$, but $\mathrm{Ca}-\mathrm{O}$ distances up to $3.7 \AA$ have been listed in Table 6.

The coordination polyhedra are shown in Fig. 7 (and also in Fig. 1). The coordination shell around $\mathrm{Ca}(3)$

Table 6. Interatomic distances in the calcium-oxygen coordination polyhedra The fourfold screw operations carry atom $\mathrm{O}\left(1^{\prime \prime}\right)$ to $\mathrm{O}(1)$ to $\mathrm{O}\left(1^{\prime}\right)$

|  | $\mathrm{Ca}(1)$ |  | $\mathrm{Ca}(2)$ |  | $\mathrm{Ca}(3)$ |  | $\mathrm{Ca}(4)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| O |  | O |  | O |  | O |  |
| 14 | 2.340 Å | 5 | $2 \cdot 342$ A | 12" | 2.318 A | 12 | $2 \cdot 370$ § |
| 9 | $2 \cdot 364$ | 13 | 2.397 |  | $2 \cdot 330$ | 13 | 2.371 |
| 2 | $2 \cdot 369$ | 14 | $2 \cdot 414$ | 8 | $2 \cdot 343$ | , | 2.397 |
| 7" | $2 \cdot 409$ | 6 | 2.509 | 6 | $2 \cdot 356$ | 6 | $2 \cdot 435$ |
| 1 | $2 \cdot 416$ | $10^{\prime}$ | 2.557 | 9 | $2 \cdot 462$ | 7 | $2 \cdot 466$ |
| 10 | 2.457 | $1^{\prime}$ | 2.640 | 2 | 2.539 | $2^{\prime}$ | 2.510 |
| 4 | $2 \cdot 780$ | $8{ }^{\prime}$ | 2.668 | 10 | 2.692 | $1^{\prime}$ | $2 \cdot 794$ |
| 11 | 3.141 | 9 | 2.745 | $13^{\prime \prime}$ | 3.250 | 11 | $2 \cdot 927$ |
| 8 | $3 \cdot 522$ | 7 | $2 \cdot 855$ | 4 | $3 \cdot 407$ | 5 | 3.165 |
| $5^{\prime \prime}$ | $3 \cdot 582$ | 11 | 3.555 |  |  | 4 | $3 \cdot 431$ |
| 3 | 3.596 |  |  |  |  | 14 | 3.681 |
| $12^{\prime \prime}$ | $3 \cdot 669$ |  |  |  |  |  |  |



Fig. 7. Calcium-oxygen coordination shells viewed along a. Compare with view along $\mathbf{c}$ in Fig. 1.
can be pictured as a distorted pentagonal bipyramid, as a distorted $\mathrm{NbF}_{7}^{2-}$ configuration (Hoard, 1939) or, perhaps more descriptively, as an octahedron distorted by the approach of $\mathrm{O}(10)$ a little farther out than the others. The same description could apply to $\mathrm{Ca}(1)$ where $\mathrm{O}(4)$ is the distorting atom. Likewise the shell around $\mathrm{Ca}(4)$ can be described as an octahedron with two oxygen atoms jammed into edges, $O\left(1^{\prime}\right)$ and $\mathrm{O}(11)$. The coordination polyhedron around $\mathrm{Ca}(2)$ is a dis-
torted one; it can be pictured as a pentagonal bipyramid with an apical atom replaced by three atoms.

We are indebted to Dr James Stewart for providing us with and instructing us in the use of X-ray 63, the system of crystallographic programs for the IBM 7094 computer developed at the Universities of Washington and Maryland; with the exception of the absorption corrections, this system was used throughout this work. The assistance given by Dr J.S. Cantrell in using X-ray 63 and the many discussions with Dr A. J. Mabis are greatly appreciated. The air-brush art work of Mr H. Thomas Brown is gratefully acknowledged.

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# Molecular Structure of Xylerythrin, a Fungus Pigment 

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The molecular structure of the red fungus pigment xylerythrin has been determined by X -ray singlecrystal analysis of the bisbromoacetate derivative. The $R$ value is $106 \%$ for the 1745 independent observed reflexions.

## Introduction

Xylerythrin $\left(\mathrm{C}_{26} \mathrm{H}_{15} \mathrm{O}_{5}\right)$ is the main component of the red pigments present in wood infected by the fungus Peniophora sanguinea Bres. It has been isolated and studied chemically by Gripenberg (1965). He showed that the compound contained two hydroxyl groups, a lactone group, a conjugated carbonyl group and at
least one monosubstituted benzene ring. A singlecrystal X-ray analysis was undertaken to provide detailed information about the structure.

## Experimental

The bisbromoacetate of xylerythrin $\left(\mathrm{C}_{30} \mathrm{H}_{18} \mathrm{O}_{7} \mathrm{Br}_{2}\right)$ was prepared by Prof. Gripenberg who provided us with


[^0]:    * The estimated standard deviation is based $n n$ the mean values of the least-squares deviations of coordinates.

